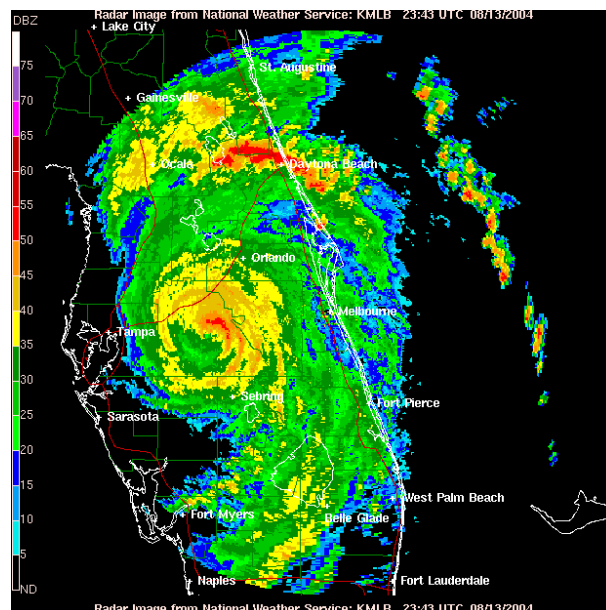


Chapter 6: The Doppler Effect

This chapter discusses two phenomena (different in cause) that affect how we perceive certain sounds. The Doppler Effect describes the change in perceived frequency when the source of a sound and its receiver are moving with respect to each other. Beats (not musical or rhythmic beats in this context) occur when two similar frequencies are played together and interfere.

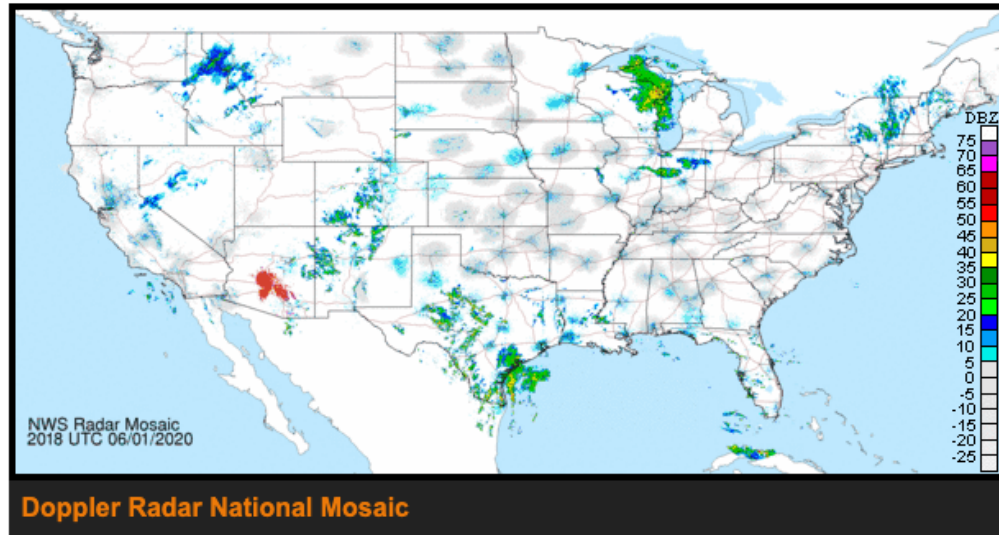
Doppler Effect

When I ask students if they have heard of the Doppler Effect before, they are quick to come up with ideas. One of the more common examples comes from the news; just about every local news station has some form of Doppler radar for the weather.



Doppler radar image of a hurricane hitting Florida ([source](#))

Both the previous picture and the newer map on the next page show a color scale that gives the viewer an idea of storm intensity. The scale is called “dBZ,” which stands for “decibels relative to Z,” which is a number of decibels (discussed in chapter 8) a radar signal gets in reflections compared to some reference level of raindrop reflection. The higher the number, the more likely severe weather will happen. The Doppler Effect discussed in this chapter is the means by which the falling precipitation speed is measured.



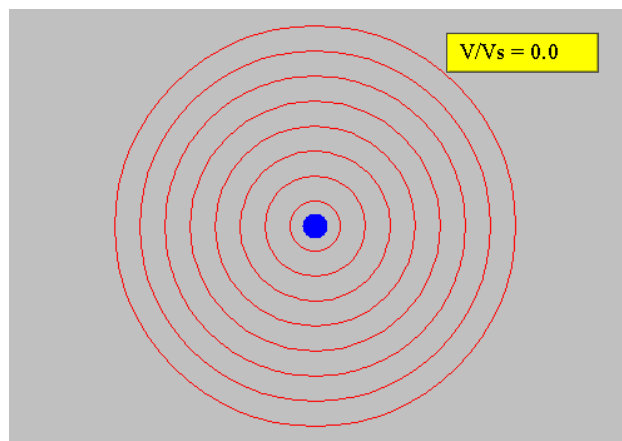
A nationwide map, courtesy of the National Weather Service.

If you have ever heard the sound of a [passing car](#) or emergency vehicle siren change, that's another example of the Doppler Effect in action. Both the weather example and the car involve waves – electromagnetic waves for the radar and sound for the car. If you listen to the passing car example, you should be able to hear that the pitch of the engines sound high as the car approaches the camera and drops as the cars move away.

The graphics I will include in the next few paragraphs can all be created using a [Doppler Effect applet](#). Try and recreate them yourself!

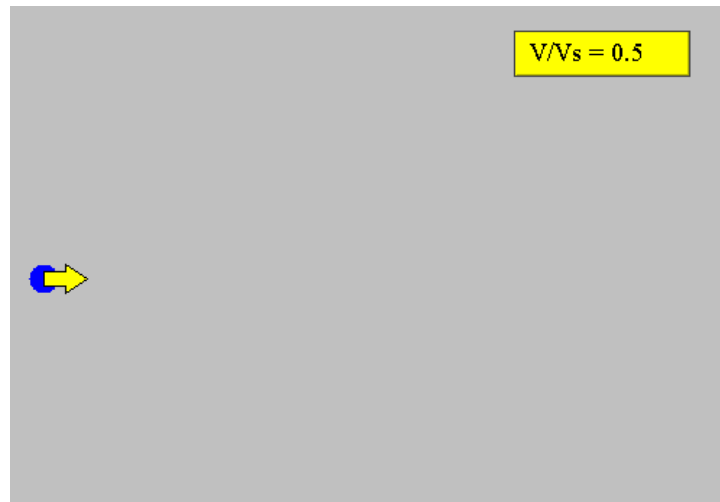
Why a moving source creates a change in perceived frequency

If you picture sound waves like ripples in a pond, a stationary source will produce uniformly spaced circular waves. To get this effect on the applet, simply click the center.

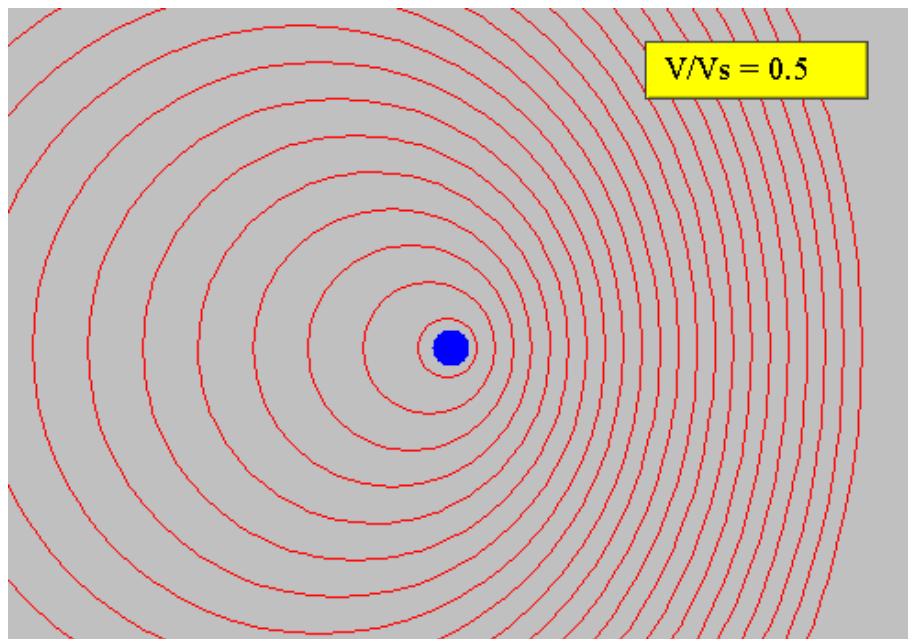


A stationary source

The real effect is visible if you click a spot on the field and drag the cursor. An arrow forms indicating the direction and speed of the source. The yellow box in the window gives the speed of the source as a fraction of the speed of the traveling waves, i.e., a value of 0.5 tells us that the source is moving at half the speed of its own waves. If you create the source on the right side and drag an arrow to the right, your screen should look like this:



If you let the animation proceed, it will look something like this:

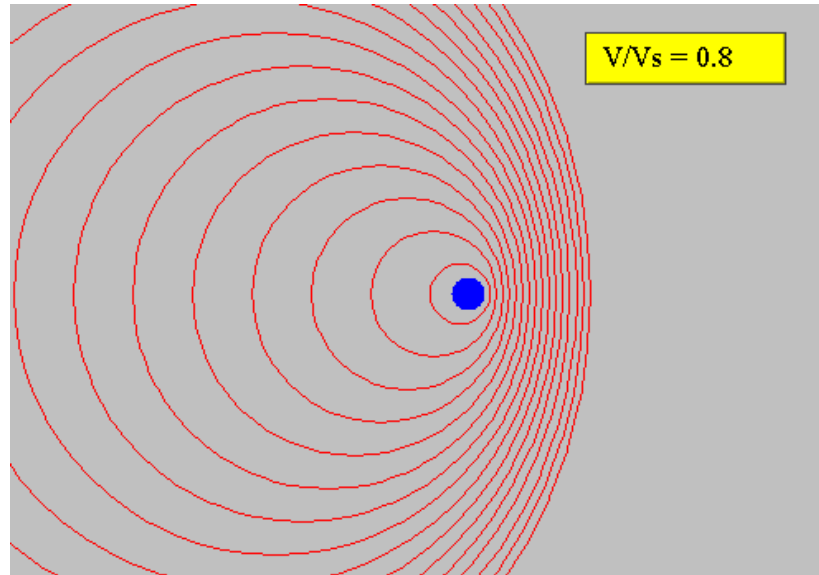


Moving source showing what happens to the waves in front and behind

It should be easy to see the effect in action here. In front of the source, the wave fronts are compressed together, so a listener would perceive more fronts arriving at his or her ears every second. Higher frequency means higher pitched. If you look behind the source, the waves are more spread out. The

separation between wave fronts is greater, the frequency one would perceive drops, and thus a lower pitch.

The effect is more dramatic the closer you get to the speed of sound:



In the case where the source speed is 4/5 the wave speed, you get very compressed (and frequently-arriving) waves in front of the source and more spread-out waves behind.

The Doppler effect for sound – mathematics

Now that we know that a moving source's perceived frequency is different than the case where it's stationary, we should see exactly how that frequency changes.

Without taking the time and space for a derivation, I will simply quote the result. If f_s is the original frequency produced by the source, v is the speed of sound in the medium, v_r is the speed of the receiver, and v_s the speed of the source, the perceived frequency f is given by:

$$f = f_s \left(\frac{v \pm v_r}{v \pm v_s} \right)$$

This may look different than most equations you're familiar with because of the plus-minus symbols. Those allow us to use this equation in multiple situations. **Use the following rule as a check:** If the source and receiver are growing nearer, the perceived frequency should be higher than the original frequency. It helps to look at simpler cases to see how to do this:

Receiver is stationary: This is the most common example in sound, as we often perceive the Doppler effect when a quickly moving vehicle passes us. If the receiver speed is zero, the Doppler frequency equation becomes:

$$f = f_s \left(\frac{v}{v \pm v_s} \right)$$

If the source is moving toward the listener, you want the perceived frequency to be higher, and the way to do that is to use the minus sign in the denominator. I always have trouble memorizing that, so I'll just check and make sure the perceived frequency f is higher if the source is approaching.

Source is stationary: If the source speed is zero, the Doppler frequency equation becomes:

$$f = f_s \left(\frac{v \pm v_r}{v} \right)$$

so in the case where the receiver is heading toward the source, you want a higher perceived frequency and you'd use the plus sign. If the receiver is moving away from the source, use the minus sign.

Equation caveats

For these equations to work, the motion of the source and receiver must be smaller than the speed of sound in the medium. If you forget this, you may be surprised to get a negative frequency once you do the math, and negative frequencies don't make much physical sense. Also, note that I just explicitly mentioned the speed of *sound*. The Doppler effect works for many kinds of waves, but the equation is not the same for light.

Also, note that the equation implies that a listener would hear just two frequencies as a source passed him or her. This is inconsistent with experience; many listeners report hearing the pitch slide continuously from high to low. That's completely true! The equation assumes the source is heading *directly* toward or away from you. In real life, a listener is often off to the side as a sound source passes, and that slight adjustment in position causes a gradual shift in frequency when the source and listener are close to one another. Astronomer John Dobson put it [like this](#): "The reason the siren slides is because it doesn't hit you."

Example problem combining multiple concepts

Suppose a police siren plays with frequency 400 Hz. Let's also suppose the car is on city streets and can't go TOO fast (maybe 60 mph), and it is 50°F outside (reasonable for the start of spring, which is when I'm writing this.) Assuming you're close to the street the police car is driving on, find the frequency you hear as the car moves (a) toward you and (b) away from you.

You may notice in the Doppler frequency equation that you need the speed of sound in air. I haven't given you that directly, but we do know the temperature. However, because I'm sneaky and tricky, I gave it to you in Fahrenheit. Convert it this way:

$$C = \frac{5}{9} (50 - 32) = \frac{5}{9} (18) = 10^\circ C$$

Then, the speed of sound can be given by the equation given in a previous chapter:

$$v = (331 + 0.6T) \text{ m/s}$$

$$v = (331 + 0.6 \times 10) \frac{\text{m}}{\text{s}} = 337 \text{ m/s}$$

We also must convert the car's speed to meters per second so that speed's units agrees with everything else we know.

$$\frac{60 \text{ mi}}{\text{hr}} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 26.8 \text{ m/s}$$

Now, we're ready to use the equation.

$$f = f_s \left(\frac{v}{v \pm v_s} \right)$$

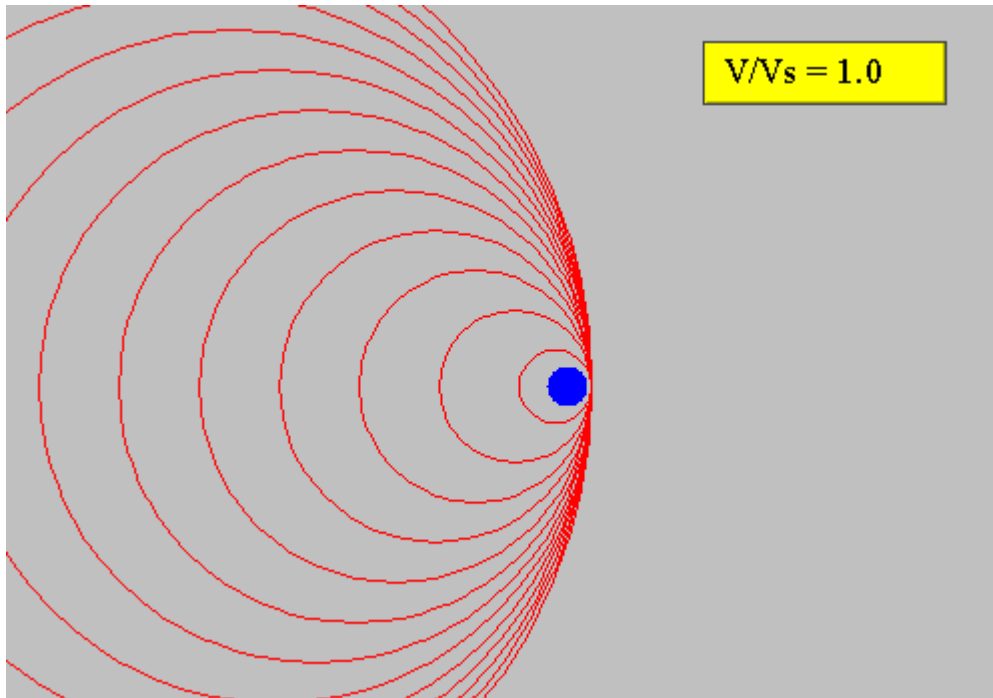
$$f = (400 \text{ Hz}) \left(\frac{337 \text{ m/s}}{337 \frac{\text{m}}{\text{s}} \pm 26.8 \text{ m/s}} \right)$$

We'll use the plus for the case when the car is moving *away* and the minus for when the car is moving *toward* you. The two answers you get, then, are 371 m/s and 435 m/s. As we'll see later when we study frequency and pitch, this is a noticeable difference.

Travel beyond the speed of sound

If a sound source approaches you at the speed of sound or faster, it will reach you before you hear it. I discussed earlier that plugging in a supersonic speed for v_s will yield mathematical results that make no sense, but let's see what happens physically.

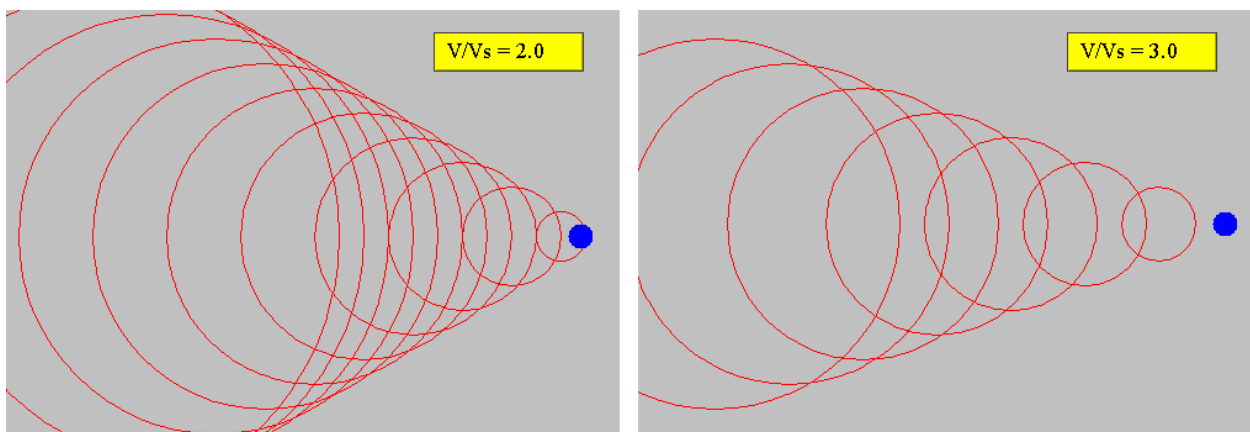
A critical moment occurs if an object breaks the sound barrier. The Java applet I linked to earlier depicts it thusly (the applet shows the motion, but I've made a still frame of it here.)



An object at the sound barrier

Notice here the waves build up at the “nose” of the moving object and create what appears to be a shock wave. You can see this pressure front as well if you study two more pictures of supersonically-moving objects on the following page. Chuck Yeager was the first person to pilot an airplane past the sound barrier (1947).

Both these objects create a cone-shaped shock wave called a *Mach cone*, named after Austrian physicist Ernst Mach. The angle of the cone narrows the faster your traveling object goes. In both cones shown below, a shock wave builds on the outside of the cone, and this will be audible as the object passes (like it would, for example, with a supersonic aircraft).



Two supersonic objects. Note their different speeds and Mach cone shapes.

While we're on the subject of Mach, the *Mach number* is the formal name given to that ratio depicted in all of my images from this applet. It is defined as:

$$\text{Mach number} = \frac{\text{speed of object moving through medium}}{\text{speed of sound in that medium}}$$

Though many commercial jets fly around Mach 0.7-0.8, many supersonic aircraft have been built. The current record (assuming we don't have a better aircraft whose design is classified) for manned airplane travel is faster than Mach 3, achieved by the SR-71 Blackbird.

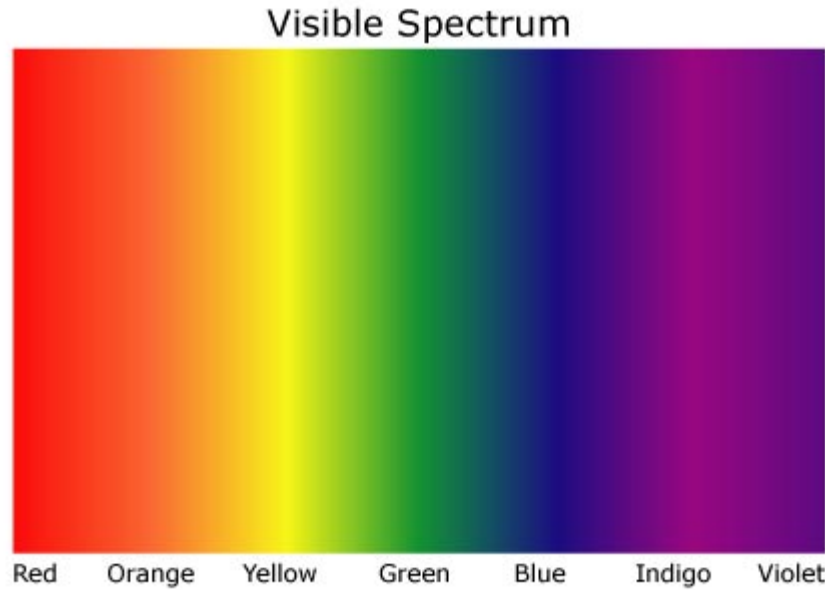


The Lockheed SR-71 Blackbird

Light and the Doppler Effect

Since the Doppler Effect is a wave phenomenon, it works for light as well. A key difference is that light travels faster than sound, so the motion required to get a noticeable Doppler effect for visible light is much faster. Doppler radar uses the Doppler effect on electromagnetic waves to detect reasonable speeds, but you wouldn't see objects noticeably change colors unless the speeds were much larger. Also, motion near the speed of light requires formulas that account for Einstein's special theory of relativity. Nothing can travel faster than light in a vacuum, so some speed relationships that work in our everyday lives have to be rethought. (Interesting aside – you can get the light version of “sonic booms,” but that's a special case: if a particle moves through a medium faster than the speed of light in that medium, it will emit an “electromagnetic boom” called Cherenkov radiation.)

Recall that the visible portion of the electromagnetic spectrum is a rainbow that may look like this:



The visible electromagnetic spectrum ([picture source](#))

The spectrum is ordered by frequency; violet light has a higher frequency and lower wavelength than red light. The “light version” of the Doppler Effect occurs much the same way the sound version does. If a source puts out light of a certain frequency and is approaching an observer, the perceived frequency will be higher and shifted toward the violet end of the spectrum. The more popular name for this shift is the *blue shift* since blue is roughly on that end of the spectrum as well. Likewise, if the source and receiver are moving apart from each other, the light will shift in frequency toward the red end of the spectrum, hence the name *red shift*.

This would be easiest to see with monochromatic (single color) light, but very few things we see travel at fast enough speeds past us to notice. Applications of this effect most often use invisible electromagnetic frequencies or multiple visible frequencies.

Much like the classical Doppler formula for sound we used earlier, the *relativistic* Doppler formula (so called because the high speeds involved mean we are bound by the laws of special relativity as devised by Einstein) dictates that the Doppler effect is more pronounced as the speed of the wave source and the wave speed become closer. In fact, the relativistic Doppler formula names a special variable, β , for the ratio of the relative velocity of source and receiver v to the speed of light in vacuum c .

$$\beta = \frac{v}{c}$$

Here, we take β and v to be positive if source and receiver are *approaching* each other. Then, the expression for the observed frequency f in terms of the original source frequency f_s is:

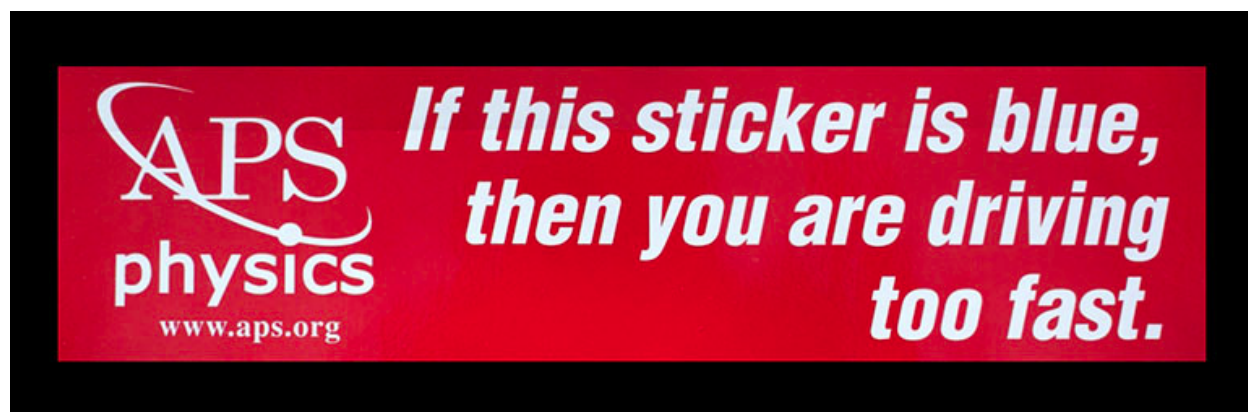
$$f = f_s \sqrt{\frac{1 + \beta}{1 - \beta}}$$

Since, for a given wave speed, frequency and wavelength are inversely proportional, we can also write:

$$\lambda_s = \lambda \sqrt{\frac{1 + \beta}{1 - \beta}}$$

Different sources have different conventions for the sign of β (for example, [here](#) and [here](#)), so it is a good idea to do a reality check with your answers. Is a receding source shifted toward higher wavelengths or lower frequencies (red shift)? Is an approaching source shifted toward lower wavelengths or higher frequencies (blue shift)? If so, you've got the signs right.

For an example, let's look at this image of a bumper sticker put out by the [American Physical Society](#):



APS Doppler sticker

Suppose, for simplicity, that the car displaying this sticker is stopped at a red light. How fast would we have to be approaching it for the sticker (and the red light) to appear blue? If you look up wavelength values anywhere reasonable, you will see that 650 nm is a red wavelength and 475 nm is a blue wavelength. Therefore, the ratio between λ_s and λ is 650/475 or about 1.37. Using the wavelength version of the relativistic Doppler equation:

$$1.37 = \sqrt{\frac{1 + \beta}{1 - \beta}}$$

Squaring both sides,

$$1.88 = \frac{1 + \beta}{1 - \beta}$$

Multiplying both sides by $1 - \beta$,

$$1.88 - 1.88\beta = 1 + \beta$$

$$0.88 = 2.88\beta$$

$$\beta = 0.305$$

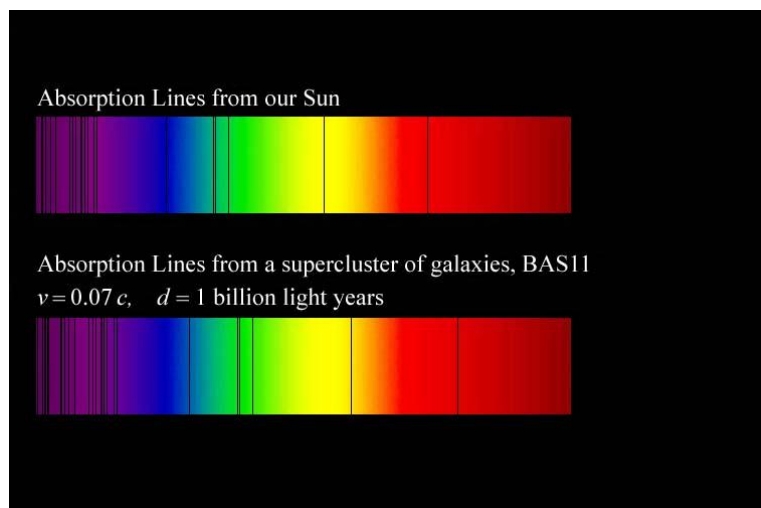
And remember, this value is the ratio of the relative speed of the objects to the speed of light. This means your car would have to approach this sticker at about 30% of the speed of light (about 91,000 km/s or about 57,000 mi/s) for it to appear blue. Definitely speeding!

Applications of the Doppler Effect for electromagnetic waves

The Doppler radar used to make the image on the first page of this chapter is of the type called *pulsed Doppler* radar, but the effect used is the same. Electromagnetic waves (often *microwave*, but radio waves are the type invoked by the name RADAR) are shot at a target, and the frequency shift upon reflection is used to find the speed of the target. The same principle is used for police radar guns.

Perhaps the most cosmically important (pun intended) applications of the Doppler Effect are found in astronomy. If you bend sunlight through a prism (or you watch drops of rain do it in a rainbow), you can see that the Sun simultaneously puts out many colors of light at once. However, a closer look reveals there are gaps in the spectrum – dark lines appear to break up the complete rainbow. Certain wavelengths (and therefore colors) of light are absent because those frequencies are absorbed by elements in the Sun's atmosphere. Hydrogen is the most common culprit. Joseph von Fraunhofer invented the spectroscope and discovered what is called the *absorption spectrum* of the sun in 1814.

Where might we use such a thing? Well, stars have similar atmospheres, so they put out similar absorption spectra. However, if a distant star is moving quickly toward or away from the sun, all of the colors from that star (and absences of color) are shifted. We can look at a distant galaxy or cluster of galaxies and determine if it's moving toward or away from us. See the picture below:



([source](#)) Doppler-shifted absorption lines

In this picture, the dark lines are all shifted to the right, which in this case is the red end of the spectrum. Since light from the distant galaxies is red-shifted, we know it is moving AWAY from us. In fact, most distant stars we can see in a spectroscope are moving away from us, and this is evidence of an expanding universe. The nearest galaxy to us, Andromeda, is a rare exception; absorption lines from there are blue-shifted.

Homework problems

1. An airplane is traveling at 80.0 m/s (180 mi/hr) and “buzzes” a house on the ground. If the plane’s engines produce a sound with frequency of 300 Hz , find the frequency heard by a stationary ground observer as the plane (a) approaches and (b) recedes from the house. Assume it is 50°F outside.
2. Consider the case of the Doppler effect in which the observer is at rest and a source of musical sound is approaching a stationary observer at twice the speed of sound in air, or 690 m/s . What does the observer hear?
3. If it is 0°C outside, how fast must a sound source approach you such that the frequency you hear is double the frequency the source is putting out? This would mean you’d hear a note one octave higher than that produced by the source. *Hint: If it bugs you that I didn’t give you either frequency, just make up a source frequency and double it for the frequency heard by the observer.*
4. The sound produced by a stationary siren of an ambulance has a frequency of 1500 Hz . Assume the air temperature is 25°C and calculate the frequency heard by the observer as the ambulance (a) approaches and (b) moves away from the observer at a speed of 30 m/s .
5. In this problem, take the speed of sound to be 343 m/s . Two horns produce identical 480 Hz sounds. However, one of the players jumps on a trolley and moves away from the listener at a speed of 8 m/s . Assuming the listener and both players lie on a straight line, calculate (a) the frequency heard by the listener and (b) the beat frequency the listener perceives.
6. A source S generates circular waves on the surface of a lake. The speed of the waves is $v_{\text{waves}} = 5.5 \text{ m/s}$, and the crest-to-crest separation is 2.3 m . You are in a small boat heading directly toward S at a constant speed of 3.3 m/s with respect to the shore. What frequency of waves do *you* observe?
7. A toy rocket moves at a speed of 242 m/s directly toward a stationary pole while emitting sound waves of frequency 1250 Hz . (a) What frequency f' is sensed by a detector attached to the pole? (b) Some of the sound reaching the pole reflects back to the rocket, which has a detector of its own. What frequency f'' does it detect?